**MATHEMATICS**
**STANDARD LEVEL**
**PAPER 2**

Thursday 6 May 2010 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

**1.** *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 2 & 4 & -3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$. *[3 marks]*

**2.** *[Maximum mark: 6]*

Consider the arithmetic sequence $3, 9, 15, \ldots, 1353$.

(a) Write down the common difference. *[1 mark]*

(b) Find the number of terms in the sequence. *[3 marks]*

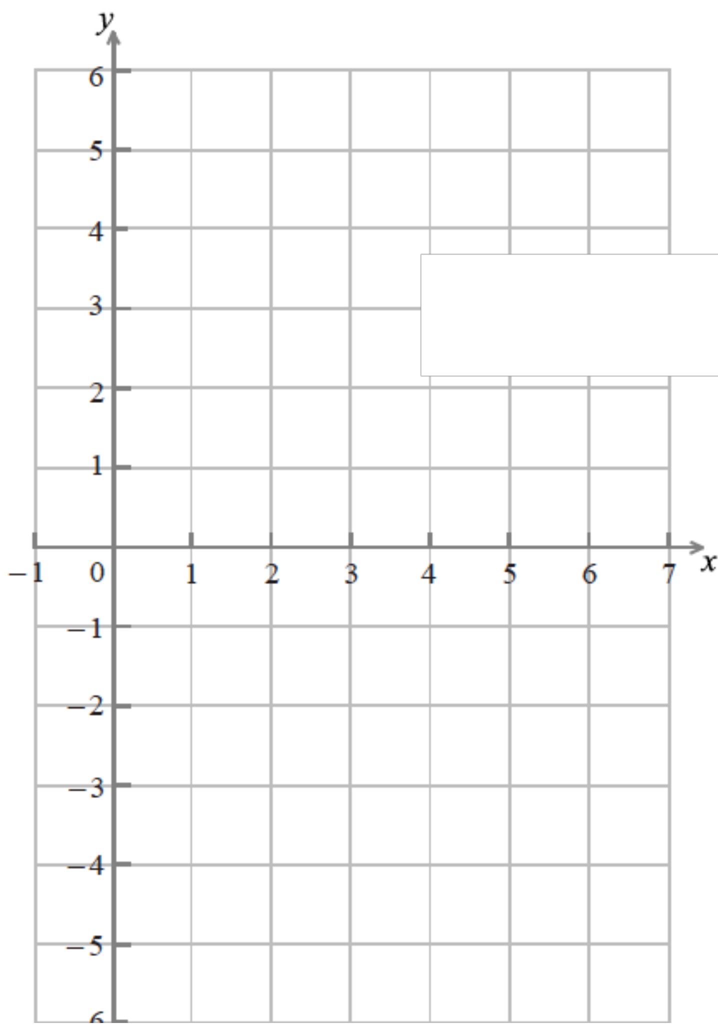
(c) Find the sum of the sequence. *[2 marks]*

**3.** *[Maximum mark: 7]*

Let $f(x) = x\cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$. *[3 marks]*

(b) On the grid below, sketch the graph of $y = f'(x)$. *[4 marks]*



**4.** *[Maximum mark: 6]*

The following frequency distribution of marks has mean 4.5.

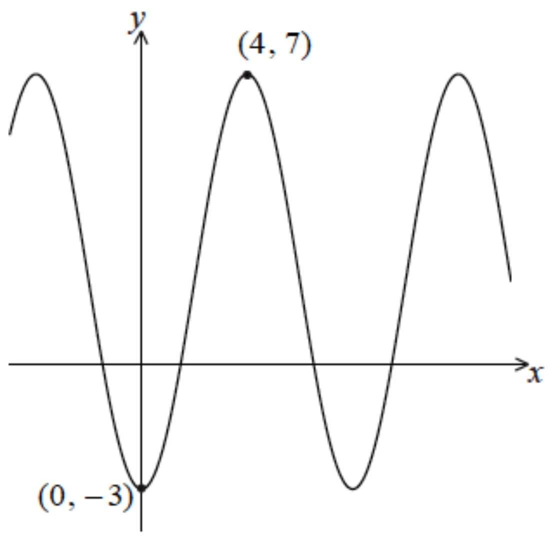
| **Mark** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Frequency** | 2 | 4 | 6 | 9 | $x$ | 9 | 4 |

(a) Find the value of $x$. *[4 marks]*

(b) Write down the standard deviation. *[2 marks]*

**5.** *[Maximum mark: 7]*

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.




There is a minimum point at $(0, -3)$ and a maximum point at $(4, 7)$.

(a) Find the value of

(i) $p$;

(ii) $q$;

(iii) $r$. *[6 marks]*

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of $k$. *[1 mark]*

**6.** *[Maximum mark: 7]*

The acceleration, $a\,\mathrm{m\,s^{-2}}$, of a particle at time $t$ seconds is given by

$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

**7.** *[Maximum mark: 7]*

Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games. *[2 marks]*

For game B, the probability that Evan wins is $p$. He plays game B seven times.

(b) Write down an expression, in terms of $p$, for the probability that he wins exactly four games. *[2 marks]*

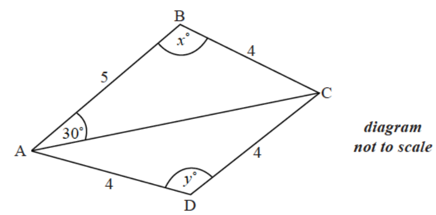
(c) Hence, find the values of $p$ such that the probability that he wins exactly four games is 0.15. *[3 marks]*

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*



**8.** *[Maximum mark: 14]*

The diagram below shows a quadrilateral ABCD with obtuse angles $\hat{ABC}$ and $\hat{ADC}$.




$AB = 5$ cm, $BC = 4$ cm, $CD = 4$ cm, $AD = 4$ cm, $B\hat{A}C = 30°$, $A\hat{B}C = x°$, $A\hat{D}C = y°$.

(a) Use the cosine rule to show that $AC = \sqrt{41 - 40\cos x}$. *[1 mark]*

(b) Use the sine rule in triangle ABC to find another expression for AC. *[2 marks]*

(c) (i) Hence, find $x$, giving your answer to two decimal places.

(ii) Find AC. *[6 marks]*

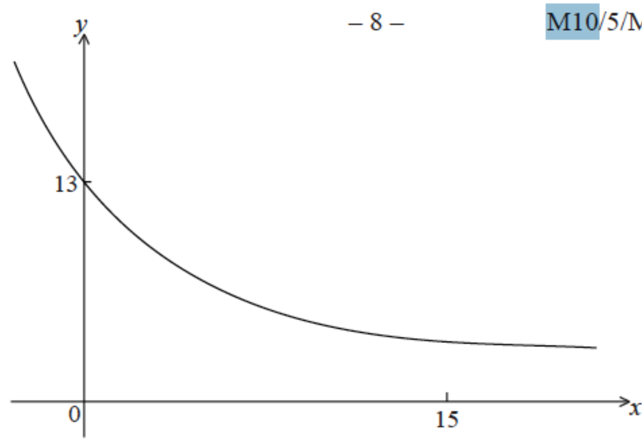
(d) (i) Find $y$.

(ii) Hence, or otherwise, find the area of triangle ACD. *[5 marks]*

**9.** *[Maximum mark: 16]*

Let $f(x) = Ae^{kx} + 3$. Part of the graph of $f$ is shown below.






The $y$-intercept is at $(0, 13)$.

(a) Show that $A = 10$. *[2 marks]*

(b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of $k$. *[3 marks]*

(c) (i) Using your value of $k$, find $f'(x)$.

(ii) Hence, explain why $f$ is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph $f$. *[5 marks]*

Let $g(x) = -x^2 + 12x - 24$.

(d) Find the area enclosed by the graphs of $f$ and $g$. *[6 marks]*

**10.** *[Maximum mark: 15]*

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg. *[2 marks]*

(b) (i) Write down the standardized value, $z$, for 68 kg.

(ii) Hence, find the standard deviation of weights. *[4 marks]*

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament. *[5 marks]*

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman. *[4 marks]*